Convolution and filtering

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Discrete convolution is defined by

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \tag{1}$$

Compute a convolution of functions $f(n)$ and $h(n)$:

$$x(n) = \begin{cases} 1, \ -5 \leq n \leq 5, \\ 0, \text{ otherwise} \end{cases} \qquad h(n) = \begin{cases} \frac{1}{4}n \ 0 \leq n \leq 4 \\ 0, \text{ otherwise} \end{cases}$$

Task 1

If the length of signal $x(n)$ is N and $h(n)$ has M samples, what is the length of $x(n) * h(n)$?

Task 2

Use script `lab2_1.m` and generate xn and hn defined with the above equations.

Task 3: Low-pass filters

In the case of image filtering, the first function $x(n)$ is image, while the second one is filtering mask – square array containing the filter coefficient. Common filtering masks are:

- Blurring filter
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} /9$$
- Low-pass filter
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 1 \end{bmatrix} /12$$

- Gaussian filter

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} /16$$

Use script `lab2_3.m` and read image `lena.jpg`. Implement one of the mentioned blurring filters. Run a convolution operation with the defined masks and answer the following questions:

1. What is the purpose of normalizing value, by which the filter coefficients are divided?
2. Explain with your own words, what low-pass filtering means.

Task 4: High-pass filters

Typical high-pass filter for images

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 9 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$

1. What is the effect of using the above filter mask on image?
2. What is undesired effect that can be noticed.

Task 5: Edge detection

- Prewitt's mask - horizontal

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

- Prewitt's mask - vertical

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

- Sobel's mask - horizontal

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

- Sobel's mask - vertical

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

- Laplace operator 1

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

- Laplace operator 2

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Using script `lab2_5.m` implement one of the Prewitt's mask, one of the Sobel's mask, and one Laplace operator. Convolve them with `lena.jpg`. What is the effect of using these masks?

Task 6 - Relief effect

A 3D-like effect can be added to an image using the follwing filter mask:

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Open and run script `relievo.m`. How the filter coefficients should be modified in order to chenge a direction of "light" on "relief".