

I. Find the inverse Laplace transform (using residues or convolution):

1. $F(s) = \frac{1}{s^3(s^2+1)},$

2. $F(s) = \frac{s}{(s^2+9)^2},$

3. $F(s) = \frac{1}{s^2(s+2)},$

4. $F(s) = \frac{1}{s^3(s-3)}.$

II. Use the Laplace transform to solve:

1. $y'' + 9y = x, \quad y(0) = 2, \quad y'(0) = 0,$

2. $y'' + 4y = 2e^{-x}, \quad y(0) = 0, \quad y'(0) = 0,$

3. $y'' + 2y' + y = e^x, \quad y(0) = 0, \quad y'(0) = -2,$

4. $y'' - 5y' + 6y = 2e^x, \quad y(0) = 1, \quad y'(0) = 1,$

5. $y'' + 4y = 2x, \quad y(0) = 0, \quad y'(0) = 1,$

6. $y'' + 4y = \cos 2x, \quad y(0) = 0, \quad y'(0) = 4,$

7. $y'' + 4y = x + \sin 2x, \quad y(0) = 0, \quad y'(0) = 0,$

8. $y'' + 4y = 1 + \sin 2x, \quad y(0) = \frac{1}{4}, \quad y'(0) = 0,$

9. $y'' + y = \cos x, \quad y(0) = 0, \quad y'(0) = 0,$

10. $y'' + y = \sin x,$

11. $y'' + 2y' + y = e^{-x},$

12. $\begin{cases} y' = z \\ z' = y + e^x + e^{-x} \end{cases}, \quad y(0) = 0, \quad z(0) = 0,$

13. $\begin{cases} y' = z + \cos x \\ z' = -y + 1 \end{cases}, \quad y(0) = 1, \quad z(0) = 1,$

14. $\begin{cases} y' + z = 1 \\ z' - y = -1 \end{cases}, \quad y(0) = 2, \quad z(0) = 1,$

15. $\begin{cases} y' = -2y - 4z + 1 + 4x \\ z' = -y + z + \frac{3}{2}x^2 \end{cases}, \quad y(0) = 0, \quad z(0) = 0,$

16. $\begin{cases} y' + 3y + z = 0 \\ z' - y + z = 0 \end{cases}, \quad y(0) = 1, \quad z(0) = 1,$

$$17. \begin{cases} y' - 2y - 4z = \cos x \\ z' + y + 2z = \sin x \end{cases}, \quad y(0) = 0, \quad z(0) = 0,$$

$$18. \begin{cases} y' = y + z - \cos x \\ z' = -2y - z + \sin x + \cos x \end{cases}, \quad y(0) = 1, \quad z(0) = 2.$$