

EXERCISES ON DIFFERENTIAL EQUATIONS

I. Solve the differential equations

1. $xy' = y \ln y$,

2. $x\sqrt{y^2-1}dx + y\sqrt{x^2-1}dy = 0$,

3. $x^2(y+1)dx + (x^3-1)(y-1)dy = 0$,

4. $(xy-x)dx + (xy+x-y-1)dy = 0$,

5. $2y' = y^2 - 1$,

6. $xy'+1 = x^3 - y'$,

7. $\sin x \cos y - y' \cos x \sin y = 0$,

8. $e^{-\frac{1}{x}}y^3 + x^2y^2y' = 0$,

9. $y^2 = xy' + y$,

10. $(x^2 - y^2)y' = xy$,

11. $(x-y)dx + (x+y)dy = 0$,

12. $x^2 + y^2 + xyy' = 0$,

13. $xyy' - x^2 + y^2 = 0$,

14. $(2\sqrt{xy} - x)y' + y = 0$,

15. $(xe^{\frac{y}{x}} + y)dx = xdy$,

16. $xy' - y \ln y + y \ln x = 0$,

17. $y' - y = x^2 - 6$,

18. $xy' - 2y = 2x^4$,

19. $y' - y = \sin 3x$,

20. $x^2 y' + xy + 1 = 0$,
21. $y' + 2xy = xe^{-x^2}$,
22. $xy' - 2y = xe^{-\frac{1}{x}}$,
23. $y' + 2y = e^{-2x}$,
24. $y' + y \operatorname{tg} x = \sin x$,
25. $(1 + x^2)y' + y = \operatorname{arctg} x$,
26. $xy' + y = y^2 \ln x$,
27. $y' + xy = x^3 y^3$,
28. $y' - 9x^2 y + 3(x^5 - x^2)\sqrt[3]{y^2} = 0$,
29. $2xyy' + x = y^2$,
30. $y' + y + y^2 \sin x = 0$,
31. $(x \cos y + \sin y)y' + x + \sin y = 0$,
25. $(y + e^x \sin y)dx + (x + e^x \cos y)dy = 0$,
32. $(2xye^{x^2} + \ln y)dx + (e^{x^2} + \frac{x}{y})dy = 0$,
33. $y'' - 2y' = 0$,
34. $2y'' - 5y' - 3y = 0$,
35. $y'' - 6y' + 9y = 0$,
36. $y'' - 2y' + 5y = 0$,
37. $y'' + 16y = 0$.

II. Solve the equation by the method of undetermined coefficients

1. $y'' - 4y' + 5y = 5x^2 - 3x + 3,$

2. $y'' + 2y' + y = x^2 e^{-x},$

3. $y'' + y = \cos x,$

4. $y'' - 2y' - 3y = e^{4x},$

5. $y'' - y = \sinh x,$

6. $y'' - 4y' + 4y = e^{2x},$

7. $y'' - 2y' + y = \cosh x.$

III. Solve by the method of variation of parameters

1. $y'' + 5y' + 6y = \frac{1}{1 + e^{2x}},$

2. $y'' - y' = \frac{2 - x}{x^3} e^x,$

3. $y'' + y = \operatorname{tg} x,$

4. $y'' + 4y = \frac{1}{\cos 2x},$

5. $y'' - y = \frac{e^x - e^{-x}}{e^x + e^{-x}},$

6. $y'' + y = \sin^2 x.$